

2.7 - Linear Programming

LT: I can use linear programming procedures to solve applications.

Example

The profit on each set of CD's that are manufactured by MusicMan, Inc., is \$8. The profit on a single CD is \$2. Machines A and B are used to produce both types of CD's. Each set takes nine minutes on Machine A and three minutes on Machine B. Each single takes one minute on Machine A and one minute on Machine B. If machine A is run for 54 minutes and Machine B is run for 42 minutes, determine the combination of CD's that can be manufactured during the time period that most effectively generates profit within the given constraints.

Linear Programming Procedure

1. Define variables.
2. Write the constraints as a system of inequalities.
3. Graph the system and find the coordinates of the vertices of the polygon formed.
4. Write an expression whose value is to be maximized or minimized.
5. Substitute values from the coordinates of the vertices into the expression.
6. Select the greatest or least result.

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$x = \#$ of sets of CDs

$y = \#$ of single CDs

$$\begin{cases} \textcircled{1} \begin{cases} 9x + 1y \leq 54 & \text{Time on Machine A} \\ 3x + 1y \leq 42 & \text{Time on Machine B} \end{cases} \end{cases}$$

$$\begin{cases} \textcircled{3} x \geq 0 \\ \textcircled{4} y \geq 0 \end{cases}$$

$$\textcircled{1} 9x + 1y \leq 54 \quad \textcircled{2} 3x + 1y \leq 42$$

$$P(x, y) = 8x + 2y$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 54 \\ 6 & 0 \end{array} \quad \frac{9x = 54}{9} = \frac{6}{1}$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 42 \\ 14 & 0 \end{array}$$

$$P(0, 42) = 8(0) + 2(42) = 84 \quad \text{Pt of intersection of } \textcircled{1} \text{ \& } \textcircled{2}$$

$$P(2, 36) = 8(2) + 2(36) = 88$$

$$P(0, 0) = 8(0) + 2(0) = 0$$

$$P(6, 0) = 8(6) + 2(0) = 48$$

$$\begin{bmatrix} 9 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 54 \\ 42 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 2 \\ 36 \end{bmatrix}$$

\therefore The profit is maximized at \$88 when 2 sets & 36 singles are produced.